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Possible Second Class Axial Coupling and Radiative Muon Absorption in Liquid Hydrogen.

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Radiative muon absorption has already been discussed by different authors (1). We have already examined (2) the possibility of detecting from observation of radiative muon absorption on nuclei possible terms in the weak axial current with second class behaviour under G-transformation (3). In this note we discuss radiative muon absorption in liquid hydrogen including a possible term in the axial current with second-class behaviour under G. The conserved vector current hypothesis, recently confirmed by experiment (4), implies that the vector current has first-class behaviour under G. A recent calculation of the pμp molecular ion (5) has led to substantial agreement between the measured capture rate (6) and the theoretical prediction based on the standard choice for the weak couplings. Such an agreement substantially encourages our faith that also the axial current has pure second-class behaviour. However, no direct evidence exists so far against second-class axial terms. Tests for such terms were proposed by WEINBERG (3). Among such tests, the comparison for the mirror transitions ^{12}B, ^{12}N $\to$ ^{12}C (with $\Delta J = 1$, no) indicates that the ft values for the two transitions differ by $(14\pm2.5)\%$ or $(16\pm3)\%$ (7).

(1) K. HUANG, C. N. YANG and T. D. LEE: *Phys. Rev.*, **108**, 1340 (1957); J. BERNSTEIN: *Phys. Rev.*, **115**, 694 (1959); G. K. MANACHER and L. WOLFENSTEIN: *Phys. Rev.*, **116**, 782 (1959); H. PRIMAKOFF: *Rev. Mod. Phys.*, **31**, 802 (1959); G. A. LOBOV and I. S. SHAPIRO: *Soviet Phys. JETP*, **43**, 1821 (1961); H. ROOD and H. TOLHOEK: *Phys. Lett.*, **6**, 121 (1963); G. OLAT: to be published.

(2) E. BORCHI and R. GATTO: *Nuovo Cimento*, **33**, 1472 (1964).

(3) S. WEINBERG: *Phys. Rev.*, **112**, 1375 (1958).

(4) T. MAYER-KUCKUCK and F. C. MICHEL: *Phys. Rev.*, **127**, 545 (1962); Y. K. LEE, L. W. MO and C. WU: *Phys. Rev. Lett.*, **10**, 253 (1963); N. W. GLASS and R. W. PETERSON: *Phys. Rev.*, **130**, 299 (1963).

(5) W. R. WESSEL and P. PHILLIPSON: *Phys. Rev. Lett.*, **13**, 23 (1964).

(6) BLEAR *et al.*: *Phys. Rev. Lett.*, **8**, 288 (1962); R. HILDEBRAND and DOEDE: *Proc. of of the 1962 Conference on High Energy Physics at Geneva* (1962), p. 418; BERTOLINI *et al.*: *Proc. of the 1962 Conference on High Energy Physics at Geneva* (1962), p. 421; ROTHBERG *et al.*: *Phys. Rev.* **132**, 2664 (1963).

(7) T. K. FISHER: *Phys. Rev.*, **130**, 2388 (1963); R. W. PETERSON and N. W. GLASS: *Phys. Rev.*, **130**, 292 (1963).

Such difference could be due to the possible second-class term of the axial current. The matrix elements of the two mirror transitions could however differ also because of electromagnetic effects, which are difficult to estimate. The analysis by HUFFAKER and GREULING (8) is based on the assumption of equal matrix elements and indicates a large second-class tensor contribution to the axial current together with a large pseudoscalar. The data on radiative muon absorption in ^{40}Ca (9) do not agree well with the results of such analysis (2).

The calculations are made for the $V-A$ theory with conserved vector current. The matrix elements of the vector current V_λ, and of the axial-vector current, A_λ, are written as

(1) $$V_\lambda = \bar{u}_n(C_V\gamma_\lambda - iC_M\sigma_{\lambda\mu}q_\mu)u_p\,,$$

(1′) $$A_\lambda = \bar{u}_n(C_V\gamma_5\gamma_\lambda + C_p\gamma_5 q_\lambda - iC_T\gamma_5\sigma_{\lambda\mu}q_\mu)u_p\,,$$

where u_n and u_p are the neutron and proton spinors; $q_\lambda = (p-n)_\lambda$ with p and n denoting the neutron and proton momenta respectively; the form factors C depend on q^2 and are assumed real (*). The q^2-dependence is neglected in C_V, C_M, C_A and C_T whereas C_P is assumed to originate from one pion exchange (see Fig. 1) and is taken proportional to the pion propagator $(m_\pi^2 - q^2)^{-1}$ (10). The term proportional to C_T in (1′) is usually dismissed on the basis that it has a different behaviour under $G = C\exp[i\pi I_2]$ from the other terms of A_λ (3). We discuss radiative muon absorption in hydrogen using exactly the same model that we have used in discussing the radiative process in nuclei (2). We derive the amplitude for $\mu^- + p \to n + \nu + \gamma$ under the assumptions that: i) The terms proportional to C_A, C_T and C_M in the radiative amplitude originate from internal bremsstrahlung graphs (including the contributions from the anomalous moments), or from catastrophic graphs obtained through the substitution $q_\lambda \to q_\lambda - eA_\lambda$ in the terms proportional to C_A, C_T, C_V and C_M in the effective hamiltonian; ii) The terms proportional to C_P are obtained by taking into account all the different possibilities of photon emission from the one-pion-exchange graph of Fig. 1. (The pion-lepton vertex is proportional to the virtual pion momentum and is assumed to give rise to an electromagnetic coupling through $q_\lambda \to q_\lambda - eA_\lambda$.) A similar model is also used in the works by MANACHER (11) and by LUYTEN, ROOD and TOLHOEK (12). The coupling constants for muon absorption are taken as follows: $C_V = 0.97C_V^{(\beta)}$, $C_A = C_A^{(\beta)} = -1.25C_V^{(\beta)}$, $\mu C_M = 3.71C_V(\mu/2m)$, $\mu C_P = 8C_A^{(\beta)}$ and (case I) $\mu C_T = 0$, (case II) $\mu C_T = 2(\mu/m)C_A$, (case III) $\mu C_T = 4(\mu/m)C_A$, (case IV) $\mu C_T = C_A$. We have called μ the muon's mass and $C_V^{(\beta)}$ the vector β-decay constant. Calculation of the effective hamiltonian for $\mu^- + p \to + \nu + \gamma$ in our model

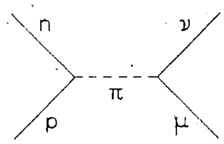

Fig. 1.

(8) J. HUFFAKER and E. GREULING: *Phys. Rev.*, **132**, 738 (1963).

(9) M. CONVERSI, R. DIEBOLD and L. DI BELLA: to be published.

(*) *Note added in proof.* – A recent experiment by J. H. CHRISTENSON, J. W. CRONIN, V. L. FITCH and R. TURLAY (*Phys. Rev. Lett.*, 13, 138 (1964)), suggests the possibility of time reversal violation in weak interactions. The form factors C would then have also an imaginary part. It is possible, however, that the terms violating time reversal are relatively small and may be negligible here.

(10) L. WOLFENSTEIN: *Nuovo Cimento*, 8, 882 (1958); M. GOLDBERGER and S. TREIMAN: *Phys. Rev.*, **111**, 354 (1958).

(11) G. K. MANACHER: Carnegie Inst. of Techn. Report NYO 9284 (1961); Our work is a direct generalization of Manacher's calculations.

(12) J. R. LUYTEN, H. P. C. ROOD and H. A. TOLHOEK: *Nucl. Phys.*, **41**, 236 (1963).

gives (for s-state absorption)

$$H_{\text{eff}}^{(R,L)} = \frac{e}{(2\pi)^2}\frac{1}{\sqrt{2K}}\frac{1}{\pi a_0^3}(1-\gamma_5^L)(\mathscr{E}_{R,L}+\gamma_5^N \mathscr{O}_{R,L}),$$

where K is the photon energy; a_0 the muonic Bohr radius (with reduced mass); the upper indices L, N indicate whether the matrix acts on the nucleons or on the leptons; R and L are for emission of right-handed or left-handed circularly polarized photons; $\mathscr{E}_{R,L}$ and $\mathscr{O}_{R,L}$ are given by

$$\begin{aligned}(3)\qquad \mathscr{E}_{R,L} = &-\frac{1}{\mu}[C_V+\boldsymbol{\sigma}^L\cdot\boldsymbol{\sigma}^N C_A - i\boldsymbol{\sigma}^L\wedge\boldsymbol{\sigma}^N\cdot(\boldsymbol{\nu}+\boldsymbol{k})C_M+\boldsymbol{\sigma}^N\cdot(\boldsymbol{\nu}+\boldsymbol{k})C_T]\cdot\\ &\cdot\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}\frac{\lambda+1}{2}+i\boldsymbol{\sigma}^L\wedge\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}\,C_M-\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}\,C_T-\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}\,C_S\frac{1-\lambda}{2}+\\ &+\frac{1}{2m}[(\lambda-\boldsymbol{\sigma}^L\cdot\boldsymbol{\sigma}^N)(C_V-\lambda C_A)+(-i\lambda\boldsymbol{\sigma}^L\wedge\boldsymbol{\sigma}^N\cdot\boldsymbol{\nu}-k\boldsymbol{\sigma}^L\cdot\boldsymbol{\sigma}^N+\\ &+\boldsymbol{\sigma}^N\cdot\boldsymbol{\nu})(C_M+\lambda C_T)-\mu C_p^{(N)}-\lambda\mu C_S]\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}+\frac{\lambda}{2m}(\mu_p-\mu_n)\cdot\\ &\cdot[C_V\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}+C_A\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}-iC_M\boldsymbol{\sigma}^L\wedge\boldsymbol{\epsilon}\cdot\boldsymbol{\nu}+C_T(-k\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}+\boldsymbol{\nu}\cdot\boldsymbol{\epsilon})+\\ &-\mu C_S\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}]+\frac{1}{2m}\frac{2\mu C_p^{(N)}}{(\boldsymbol{\nu}+\boldsymbol{k})^2+m_\pi^2}(\boldsymbol{\epsilon}\cdot\boldsymbol{\nu})(\boldsymbol{\sigma}_N\cdot\boldsymbol{\nu}),\end{aligned}$$

$$\begin{aligned}(3')\qquad \mathscr{O}_{R,L} = &-\frac{1}{\mu}[\boldsymbol{\sigma}^L\cdot\boldsymbol{\sigma}^N C_V+C_A+\boldsymbol{\sigma}^N\cdot((\boldsymbol{\nu}+\boldsymbol{k})C_M-i\boldsymbol{\sigma}^L\wedge\boldsymbol{\sigma}^N(\boldsymbol{\nu}+\boldsymbol{k})C_T]\cdot\\ &\cdot\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}\frac{\lambda+1}{2}+i\boldsymbol{\sigma}^L\wedge\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}\,C_T-\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}\,C_M-\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}\,C_p^{(L)}\frac{1-\lambda}{2}+\\ &+\frac{1}{2m}[(\lambda-\boldsymbol{\sigma}^L\cdot\boldsymbol{\sigma}^N)(-\lambda C_V+C_A)+(-i\boldsymbol{\sigma}^L\wedge\boldsymbol{\sigma}^N\cdot\boldsymbol{\nu}-\lambda k\boldsymbol{\sigma}^L\cdot\boldsymbol{\sigma}^N+\\ &+\lambda\boldsymbol{\sigma}^N\cdot\boldsymbol{\nu})(C_M+\lambda C_T)-\lambda\mu C_p^{(N)}-\mu C_S]\,\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}+\frac{\lambda}{2m}(\mu_p-\mu_n)\cdot\\ &\cdot[C_V\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}+C_A\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}+C_M(-k\boldsymbol{\sigma}^L\cdot\boldsymbol{\epsilon}+\boldsymbol{\nu}\cdot\boldsymbol{\epsilon})-iC_T\boldsymbol{\sigma}^L\wedge\boldsymbol{\epsilon}\cdot\boldsymbol{\nu}-\mu C_p^{(N)}\boldsymbol{\sigma}^N\cdot\boldsymbol{\epsilon}].\end{aligned}$$

In (3) and (3′) we have also included possible contributions from a scalar term, of the form $C_S q_\lambda$, in the vector current V_λ. Such a term will however be ignored in the following, on the basis of the present evidence in favor of the conserved vector current hypothesis [4]. In (3) and (3′) $\lambda=+1$ or -1 for R and L respectively; $C_p^{(N)}$ and $C_p^{(L)}$ are given by

$$(4)\qquad C_p^{(L)} = C_p\frac{\mu^2+m_\pi^2}{(\boldsymbol{\nu}+\boldsymbol{k})^2+m_\pi^2},\qquad C_p^{(N)} = C_p\frac{\mu^2+m_\pi^2}{\boldsymbol{\nu}^2-\boldsymbol{k}^2+m_\pi^2},$$

and $\boldsymbol{\nu}$ is the neutrino momentum.

We follow Weinberg's analysis of μ-capture [13] assuming that in liquid hydrogen

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[13] S. WEINBERG: *Phys. Rev. Lett.*, 8, 575 (1960).

the absorption (ordinary or radiative) occurs from the ortho-state of the muonic hydrogen molecular ion. The experimentally observed molecular absorption rate is given ([13]) by

$$\Lambda_{\text{mol}} = 2\gamma\{(\xi - \tfrac{1}{3})\Lambda_0 + (\tfrac{4}{3} - \xi)\}\,, \tag{5}$$

where Λ_0 is the atomic absorption rate in hyperfine singlet state, and Λ is the absorption rate from unpolarized proton. According to WEINBERG ([8]) $\gamma = 0.582$ and $\frac{1}{2} \leqslant \xi \leqslant 1$. We shall use however the more recent result $\gamma = 0.500$ ([5]).

In Table I we report, for the four choices considered for C_T, the values of Λ, Λ_0, Λ_0/Λ, and of Λ_{mol} and of the ratio R_{mol} of radiative to ordinary absorption in the molecular ion for the extreme choices $\xi = \frac{1}{2}$ and $\xi = 1$.

TABLE I. – *Ordinary and radiative absorption rates in liquid hydrogen.*

Case	μC_T	$\Lambda \Big/ \left(\dfrac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3}\right)$	$\Lambda_0 \Big/ \left(\dfrac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3}\right)$	Λ_0/Λ
I	0	4.80	18.50	3.85
II	$2(\mu j m)C_A$	4.62	16.65	3.62
III	$4(\mu/m)C_A$	4.56	15	3.30
IV	C_A	4.95	11.40	2.30

Case	$\Lambda_{\text{mol}} \Big/ \left(\dfrac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3}\right)$ $\xi = \frac{1}{2}$	$\Lambda_{\text{mol}} \Big/ \left(\dfrac{\mu^2 C_V^{(\beta)2}}{\pi^2 a_0^3}\right)$ $\xi = 1$	$R_{\text{mol}} \cdot 10^4$ $\xi = \frac{1}{2}$	$R_{\text{mol}} \cdot 10^4$ $\xi = 1$
I	7.05	13.6	2.18	0.45
II	6.65	12.50	2.52	0.63
III	6.29	10.53	2.92	0.84
IV	6.05	9.13	4.15	1.82

A general feature to be noted is the decrease of both Λ and Λ_0 with the increase of C_T. We note that $\Lambda_0/\Lambda = 4$ would correspond to no absorption from the atomic hyperfine triplet state. Similarly, $\Lambda_0/\Lambda = 1$ would correspond to equal absorption from the triplet and from the singlet. For small C_T the absorption is thus mostly from the singlet — a well-known fact. For radiative absorption the situation is different: absorption from the singlet is small in the absence of C_T. The photon spectrum from radiative absorption in muonic ortho-hydrogen molecular ion can be written as

$$N_{\text{mol}}(x) = 2\gamma\{(\xi - \tfrac{1}{3})N_0(x) + (\tfrac{4}{3} - \xi)N(x)\}\,, \tag{6}$$

where $x = K/\mu$, and $N_0(x)$ and $N(x)$ are the spectra for singlet absorption and for absorption by an unpolarized proton. The spectra $N_0(x)$ and $N(x)$ have been evaluated from (2), (3) and (3′) by averaging the squared matrix element over the singlet μ^--p state ($|\text{singlet}\rangle = (2)^{-\frac{1}{2}}[\alpha^N\beta^L - \alpha^L\beta^N]$) and on all four states respectively.